RADIATIVE TRANSFER IN SPECTRAL LINES WITH SELF-ABSORPTION

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In calculations of radiative heat transfer in gas layers with nonuniform temperature distributions one often must taken into account the strong spectral dependence of the absorption coefficient. The analytical relations governing the problem are quite complicated, and even in the case of the monochromatic specific intensity these relations take a simple form only for the plane problem. Therefore there arises the need for approximate methods of calculation, which, in particular, may be based on the differential approximation for radiative heat transfer. In this paper we consider two problems—we compare the differential approximation with the exact solution for the case of a plane layer, and we propose a method of calculating the radiative heat transfer in spectral lines.

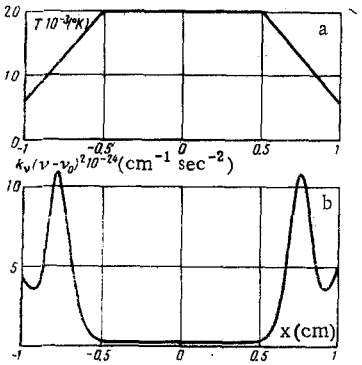

Fig. 1

Notation

k_ν	monochromatic absorption coefficient per unit volume [cm^{-1}]
ν	frequency [sec^{-1}]
ν_0	frequency at line center [sec^{-1}]
c	speed of light [cm · sec^{-1}]
σ	Stefan-Boltzmann constant
T	absolute temperature [°K]
$\mathbf{q}_\nu$	monochromatic radiative flux vector (per unit frequency interval) [erg/cm^2 · sec · sec^{-1}]
u_ν, u_ν°	monochromatic and equilibrium radiation energy density [erg · cm^{-3}]
Λ	extrapolated length

$$q_{\Delta\nu}=\int_{\Delta\nu} q_\nu d\nu \left[\text{erg/cm}^2\ \text{sec}\right], \qquad \varepsilon_i=\frac{cu_{\nu_i}^\circ \Delta\nu_i}{4\sigma T^4}.$$

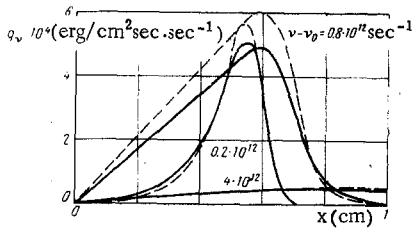

Fig. 2

1. We consider a typical case of radiative transfer in spectral lines (electric arcs [1], high-temperature gas flows, etc.). The model is a plane layer of argon with the temperature distribution shown in Fig. 1a. We assume local thermodynamic equilibrium. The radiative transfer is to be calculated for the strong resonance line 3P–4S, with oscillator strength 0.2. The broadening of this line is mainly due to resonance and to the quadratic Stark effect. In this case most of the energy is transferred relatively far from the center of the line, where the line profile has a dispersive character. Figure 1b shows the profile of the quantity $k_\nu(\nu-\nu_0)^2$ as a function of the thickness of the gas layer. In the frequency range between $\nu_1-\nu_0 = 0.25\cdot10^{12}$ sec^{-1} and $\nu_2-\nu_0 = 4\cdot10^{12}$ sec^{-1}, in which most of the energy is transferred, the absorption coefficient varies with frequency by a factor of 250 and with temperature by a factor of 40.

Exact expressions for the radiative flux vector $\mathbf{q}_\nu$ and for the divergence div $\mathbf{q}_\nu$ can be written in terms of integrals [2, 3]. To obtain an approximate solution we use the differential approximation (first term of an expansion in spherical harmonics [4]), represented by the equations

$$\frac{1}{k_\nu}\operatorname{div} q_\nu = cu_\nu^\circ - cu_\nu, \qquad q_\nu = -\frac{1}{3k_\nu}\operatorname{grad} cu_\nu. \quad (1.1)$$

In the absence of external radiation incident on the gas layer, the boundary condition is

$$cu_\nu = \frac{\Lambda}{k_\nu}\frac{\partial cu_\nu}{\partial n}.$$

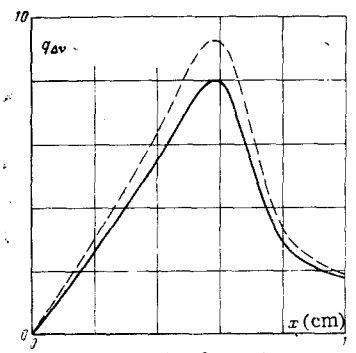

Fig. 3

The differential approximation takes into account the integral nature of radiative transfer. In the limit $k_\nu \to \infty$ the equation for $\mathbf{q}_\nu$ reduces to the diffusion approximation; in the limit $k_\nu \to 0$ it reduces to the correct optically thin formulation; in the intermediate range of opacities it constitutes an approximation, whose accuracy can be estimated by comparison with exact solutions of the equation of radiative transfer. In the case of a plane layer, one can also use the approximation (1.1) to derive integral expressions for the variable $\mathbf{q}_\nu$ and div $\mathbf{q}_\nu$. It should be noted that the half-range approximation with the average cosine equal to one-half [3], does not lead to better results.

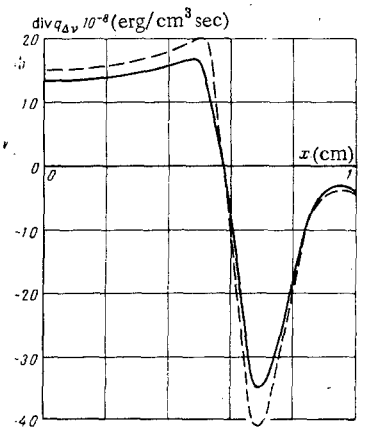

Fig. 4

Figure 2 shows the profiles of q_ν across the layer for different frequencies, as calculated by the exact formulation (solid lines) and by the differential approximation (broken lines). Comparing these results one sees that in the intermediate range of opacities the differential approximation (1.1) results in a maximum error of 20%. In the range of high k_ν the error may be higher, but this is of little importance in view of the fact that the values q_ν and div q_ν themselves tend to zero. In the range of low k_ν the error decreases, and for $k_\nu \to 0$ the approximation reduces to the correct optically thin formulation. Figure 3 shows the profile of $q_{\Delta\nu}$, where the range of integration is $\Delta\nu = \nu_2 - \nu_1 = 3.75 \cdot 10^{12}$ sec^{-1}. It can be seen that the error in the integrated radiative flux does not exceed 17%, and the error in div $q_{\Delta\nu}$ (Fig. 4) is of the same order. Thus, it appears that for most practical calculations the differential approximation yields acceptable accuracy.

2. The spectral interval under the broadened line can be divided into three ranges [1]: (1) the range of high values of the absorption coefficients, in which one may use the diffusion approximation; (2) the range of low values of the absorption coefficient, in which one may use the optically thin formulation; and (3) the range of intermediate values of the absorption coefficient, in which one may use the differential approximation. As has been noted elsewhere [5, 6], problems involving frequency-dependent absorption coefficients may be solved by breaking up the spectrum into a number of intervals with nearly constant coefficients. It has also been noted, however, that the number of such intervals would be unrealistically large [7].

It would be unrealistic to attempt to solve the heat-conduction equation for each value of the frequency. Therefore there arises

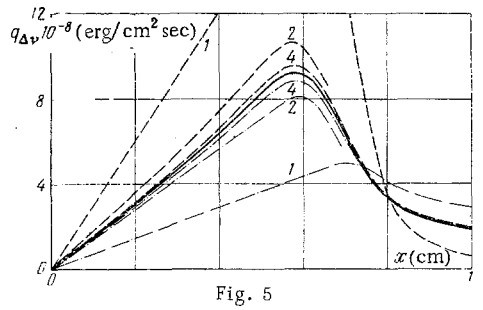

Fig. 5

the need for an appropriate method of averaging the absorption coefficient. In the present case we subdivide range (3) of the argon line under consideration into a number of subintervals, within each of which the absorption coefficient varies by the same factor. In each subinterval the absorption coefficient was averaged by two methods, viz.,

$$k' = \frac{1}{\Delta\nu}\int_{\Delta\nu} k_\nu d\nu, \qquad k'' = \nu\Delta\nu\left(\int_{\Delta\nu}\frac{1}{k_\nu}d\nu\right)^{-1}. \tag{2.1}$$

Figure 5 shows the integrated radiative flux $q'_{\Delta\nu}$ (broken lines) and $q''_{\Delta\nu}$ (dot-and-dash lines) obtained by these two methods.

The results presented in this section were obtained using the differential approximation. The number next to each curve denotes the number of subintervals in range (3). The solid line represents the values of $q_{\Delta\nu}$ obtained by integration over the frequency interval $\Delta\nu$. It can be seen that the curves based on (2.1) lie on both sides of $q_{\Delta\nu}$, and converge to $q_{\Delta\nu}$ with increasing number of subintervals. Even when the range (3) (in which practically all the energy of the line is transferred) is divided into subintervals in which the absorption coefficient varies by a factor of four, the values of $q'_{\Delta\nu}$ and $q''_{\Delta\nu}$ differ from $q_{\Delta\nu}$ by about 10%.

It should be noted that in the model considered here range (3) corresponds to the dispersive asymptotic profile. It can be shown (using (2.1)), that a different power function in the dispersion asymptote would not increase the deviation of $q'_{\Delta\nu}$ and $q''_{\Delta\nu}$ from $q_{\Delta\nu}$. Thus, dividing the energetically significant part of the spectral line into a small number of subintervals (four in the present case), one obtains acceptable accuracy with either one of the two averaging methods (2.1). Using the geometric mean of the two averages k' and k",

$$\langle k\rangle = \left(\int_{\Delta\nu} k_\nu d\nu \Big/ \int_{\Delta\nu}\frac{d\nu}{k_\nu}\right)^{1/2}, \tag{2.2}$$

one obtains even better results.

Figure 6 shows the values of $q_{\Delta\nu}$ calculated on the basis of (2.2). The number next to broken line denotes the number of subintervals. Even if one calculates the average over the whole region (3) in one step (k_ν varies by a factor of more than 250), the error in the range of high temperatures (which is usually the most important) is small. When one uses two subintervals the error does not exceed 20%. In the case of three subintervals (circles) there is practically no difference between the approximate and the exact calculation.

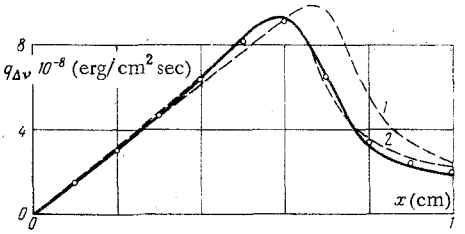

Fig. 6

Figure 7 shows the values of div $q_{\Delta\nu}$ calculated on the basis of (2.2). The broken lines represent values obtained using one and two subintervals, and the circles represent values obtained using three subintervals.

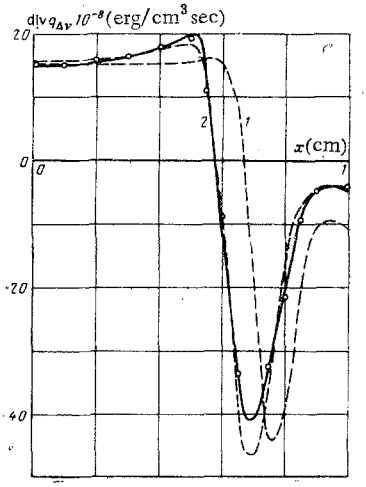

Fig. 7

Problems involving several self-absorbing lines can be treated in the following manner: The lines are divided into groups, in each of which the absorption coefficient k_ν has the same temperature dependence. This can be done if all the transitions in a group have the same lower state (or if their lower states are close to each other) and if the line half-widths vary in the same way over the whole range of temperatures. This last requirement is quite severe, as usually different broadening mechanisms predominate in different ranges of temperature. The problem can often be simplified, however, since usually one is interested mainly in the energy transfer at high temperatures, as, e.g., in the calculation of arcs. In such cases one can usually identify a single dominant broadening mechanism for a large group of lines. In the case of argon, for example, the strongest transitions to the ground level at high temperatures are broadened by the quadratic Stark effect, so that all these lines can be considered as one group. Under these conditions, all lines belonging to a single group can be accounted for by the same average

absorption coefficients. The energy transferred in corresponding ranges of the absorption coefficient can then be represented by sums of apparent emissivities $\varepsilon_1 + \varepsilon_2 + \ldots$ of the corresponding spectral intervals.

Thus, a large class of problems of radiative transfer in a system of self-absorbing lines can be treated using a small number of average absorption coefficients if the averaging method described above is used.

The averaging method described here can be used both for exact and for approximate formulations of radiative transfer. In this paper we have considered the differential approximation, because this approximation leads to simple results in a more general class of cases, e.g., in the case of cylindrical symmetry [1]. Clearly, the averaging method presented here can be extended to the case of a continuous spectrum.

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